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THE FRENCH AEROSPACE LAB

# Time Schemes for Solving Maxwell's Equations in a Mesh Hybridization Strategy.

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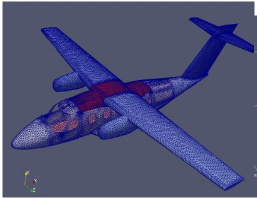
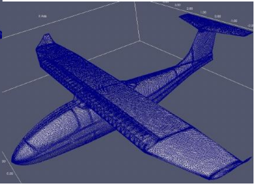
# Context and issues

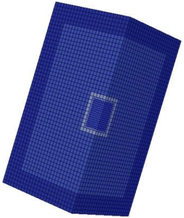
**context :** Efficient calculation of electromagnetic field values in complex structures or objects in order to qualify them. *Efficient = fast and accurate*

**Issues :**

- accurately take into account the geometric curvature of objects ;
- take into account local refinements in a multi-scale domain ;
- processing of large calculation scenes.







# Maxwell-TD : multi-domains/multi-methods approach

## software architecture

**One file to define the computational sub-domains and the hybridizations between them**

Concerning each sub-domain, we give

- a number of processus to parallal implementation ;
- a numerical method ;
- two files to define geometric and physical parameters.

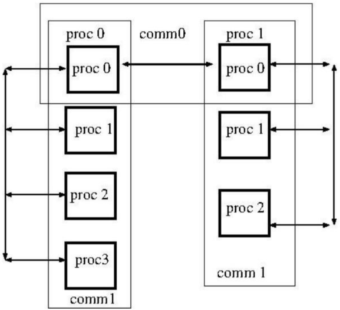
concerning each Hybridization, we give

- 2 sub-domains ;
- a type of Hybridization.

**2-level parallelism inside each sub-domain and between different sub-domains by using openMP/MPI.**

# Maxwell-TD
## software architecture

- MPI -> exchange between sub-domains ;
- OPEN/MP -> only for calculation loops.

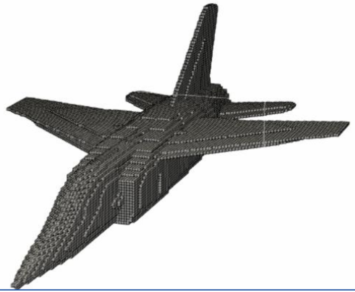


# Maxwell-TD

## mesh efficiency : cartesian meshes

**Process :**

- defines a cartesian grid $(xmin, xmax, nx) \times (ymin, ymax, ny) \times (zmin, zmax, nz)$ ;
- projection of a point cloud representing the objects, inside the grid ;
- determination meshes of the objects by the boundary cells of the point cloud.

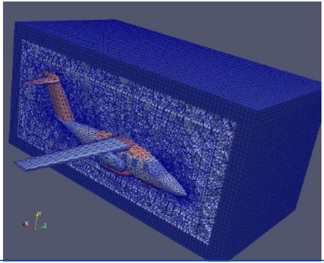


# Maxwell-TD

## mesh efficiency : NST meshes

**Process :**

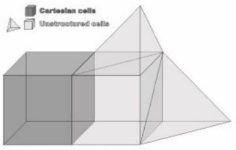
- define triangular meshes of the boundaries of sub-domains inside the computational domain ;
- generate volumic meshes of the sub-domains by using GMSH ;
- cement all the sub-domains to obain the global mesh.



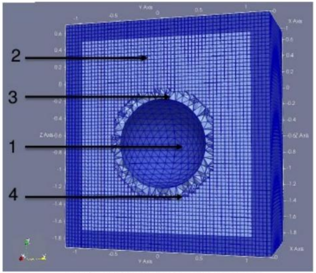
# Maxwell-TD

## mesh efficiency : hybrid ST/NST meshes : principle

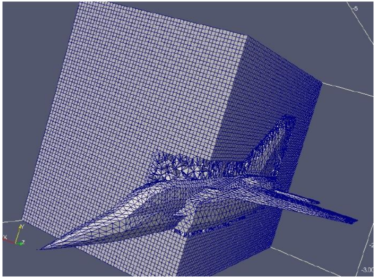
- Fast and Automatic ST-UNST Meshing Strategy
  1) Triangular surface mesh for considered object
  2) Cartesian mesh adapted to 3D computational domain
  3) UNST tetrahedral volume mesh between the surface meshes of: the object and the Cartesian/global object area boundary
  4) Merging of ST and NST meshes




Cell hybridization interface




*Hybride ST/UNST grid with generation steps*



# Maxwell-TD

## Numerical scheme

Several scheme are available inside Maxwell-TD

- FDTD : Yee's method used on cartesian meshes ;
- FVTD-NST : Finite volume on unstructured meshes ;
- FVTD-ST : Finite volume on cartesian meshes ;
- FEM : Finite element method on cartesian meshes ;
- DG : Discontinus Galerkin method on unstructured meshes ;
- MIT : Time EFIE on triangular meshes ;
- CDO : Compatible Discrete Operator on unstructured meshes ;
- SD : Spectral difference method on cartesian meshes ;
- TLM : Transmission Line Method for cables.

# Maxwell-TD

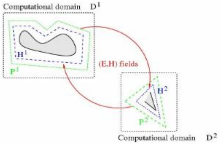
## Hybrid strategy : In progress ...

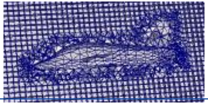
Two kinds of hybridations

- For EMC : Cables with Fields ;
- Between sub-domains in a calculation scene.

Concerning sub-domains several aspects have been studied

- By matrices of impedance ;
- Disjoints sub-domains ;
- Hybridation on mesh.



# Hybridization inside Maxwell-TD

## Numerical Aspects for FEM-DG Hybridization

### General formulation for DG method

- Computational domain $\Omega$ approximated by partition $\tau_h$ defined by cells $\mathcal{K}$
- we search a solution $(\mathbf{E},\mathbf{H}) \in \mathbf{H}^1(\tau_h) \times H^1(\tau_h)$, with $\mathbf{H}^1(\tau_h) = \{v \in \mathbf{L}^2(\Omega)\ ;\ \forall \mathcal{K} \in \tau_h, v_{|\mathcal{K}} \in \mathbf{H}^1(\mathcal{K})\}$, so that $\forall(\varphi,\psi) \in \mathbf{H}^1(\tau_h) \times \mathbf{H}^1(\tau_h)$ :

$$\left\{ \begin{array}{ll} \forall \mathcal{K} \in \tau_h, & \int_{\mathcal{K}} \varepsilon \partial_t \mathbf{E} \cdot \varphi \, dx = \int_{\mathcal{K}} \nabla \times \mathbf{H} \cdot \varphi \, dx + \gamma_{\mathcal{K}} \int_{\partial \mathcal{K}} [\![\mathbf{H} \times n]\!] \cdot \varphi \, dx \\ \forall \mathcal{K} \in \tau_h, & \int_{\mathcal{K}} \mu \partial_t \mathbf{H} \cdot \psi \, dx = -\int_{\mathcal{K}} \nabla \times \mathbf{E} \cdot \psi \, dx + \delta_{\mathcal{K}} \int_{\partial \mathcal{K}} [\![\mathbf{E} \times n]\!] \cdot \psi \, dx \end{array} \right.$$

At the bounds of cell $\mathcal{K}$ we put down a jump term

- $[\![\mathbf{E} \times n]\!] = (\mathbf{E}' - \mathbf{E}) \times n$ and $[\![\mathbf{H} \times n]\!] = (\mathbf{H}' - \mathbf{H}) \times n$, when we have $\mathcal{K}'$ so that $\partial\mathcal{K} = \mathcal{K}' \cap \mathcal{K}$, with $\mathbf{E}'$ et $\mathbf{H}'$ fields inside $\mathcal{K}'$ ;
- $[\![\mathbf{E} \times n]\!] = -\mathbf{E} \times n$ and $[\![\mathbf{H} \times n]\!] = -\mathbf{H} \times n$, when $\partial\mathcal{K} \subset \partial\Omega$.

# Hybridization inside Maxwell-TD

## Numerical Aspects for FEM-DG Hybridization

**General formulation for the FEM method**

- Computational domain $\Omega$ with a boundary $\partial\Omega$
- Search $(\mathbf{E}, \mathbf{H}) \in \mathbf{H}_0(\mathrm{rot}, \Omega) \times \mathbf{L}^2(\Omega)$ so that $\forall(\varphi, \psi) \in \mathbf{H}_0(\mathrm{rot}, \Omega) \times \mathbf{L}^2(\Omega)$ :

$$\int_\Omega \varepsilon \partial_t \mathbf{E} \cdot \varphi \, dx = \int_\Omega \mathbf{H} \cdot \nabla \times \varphi \, dx + \int_{\delta\Omega} \mathbf{H} \cdot \varphi \times n \, dx$$
$$\int_\Omega \mu \partial_t \mathbf{H} \cdot \psi \, dx = -\int_\Omega \nabla \times \mathbf{E} \cdot \psi \, dx$$

- On $\partial\Omega$, $n \times E = 0$ with $n$ the outgoing normal at $\partial\Omega$

# Hybrid FEM-GD approach

## The hybrid formulation

search $(\mathbf{E}_1(t,\cdot),\mathbf{H}_1(t,\cdot)) \in \mathbf{H}_E(\Omega_1)\times\mathbf{H}(\mathrm{rot},\Omega_1)$ and $(\mathbf{E}_2(t,\cdot),\mathbf{H}_2(t,\cdot)) \in \mathbf{H}^1(\tau_h)\times\mathbf{H}^1(\tau_h)$,
so that $\forall(\varphi_1,\psi_1)\in\mathbf{H}_E(\Omega_1)\times\mathbf{H}(\mathrm{rot},\Omega_1)$ and $\forall(\varphi_2,\psi_2)\in\mathbf{H}^1(\tau_h)\times\mathbf{H}^1(\tau_h)$

$$\int_{\Omega_1}\varepsilon\,\partial_t\mathbf{E}_1\cdot\varphi_1\,dx = \int_{\Omega_1}\mathbf{H}_1\cdot\nabla\times\varphi_1 dx + \int_\Gamma \mathbf{H}_1\cdot\varphi_1\times n_1\,ds + \alpha\int_\Gamma(\mathbf{H}_2-\mathbf{H}_1)\times n_1\cdot\varphi_1 ds$$

$$\int_{\Omega_1}\mu\,\partial_t\mathbf{H}_1\cdot\psi_1\,dx = -\int_{\Omega_1}\nabla\times\mathbf{E}_1\cdot\psi_1\,dx + \beta\int_\Gamma(\mathbf{E}_2-\mathbf{E}_1)\times n_1\cdot\psi_1\,ds$$

$$\forall_{\mathcal{K}}\in\tau_h,\ \int_{\mathcal{K}}\varepsilon\,\partial_t\mathbf{E}_2\cdot\varphi_2 dx = \int_{\mathcal{K}}\nabla\times\mathbf{H}_2\cdot\varphi_2 dx + \gamma_{\mathcal{K}}\int_{\partial\mathcal{K}\setminus\Gamma}[\![\mathbf{H}_2\times n_{\mathcal{K}}]\!]\cdot\varphi_2 ds + \gamma\int_\Gamma(\mathbf{H}_1-\mathbf{H}_2)\times n_2\cdot\varphi_2 ds$$

$$\forall\mathcal{K}\in\tau_h,\ \int_{\mathcal{K}}\mu\,\partial_t\mathbf{H}_2\cdot\psi_2\,dx = -\int_{\mathcal{K}}\nabla\times\mathbf{E}_2\cdot\psi_2\,dx + \delta_{\mathcal{K}}\int_{\partial\mathcal{K}\setminus\Gamma}[\![\mathbf{E}_2\times n_{\mathcal{K}}]\!]\cdot\psi_2 ds + \delta\int_\Gamma(\mathbf{E}_1-\mathbf{E}_2)\times n_2\cdot\psi_2 ds$$

# Hybrid FEM-GD approach

## Numerical study

$$\mathcal{E}_\Omega = \|\mathbf{E}\|^2_{0,\varepsilon,\Omega_1} + \|\mathbf{H}\|^2_{0,\mu,\Omega_1} + \|\mathbf{E}\|^2_{0,\varepsilon,\Omega_2} + \|\mathbf{H}\|^2_{0,\mu,\Omega_2}$$

Then this energy do not increase in time and its derivative in time must be equal to 0.

$$\begin{aligned}
\partial_t \mathcal{E}_\Omega =& (1+\beta-\alpha)\int_\Gamma \mathbf{H}_1 \cdot (\mathbf{E}_1 \times n_1) ds + \alpha \int_\Gamma \mathbf{H}_2 \times n_1 \cdot \mathbf{E}_1 ds + \beta \int_\Gamma \mathbf{E}_2 \times n_1 \cdot \mathbf{H}_1 ds \\
&+ (1-\gamma+\delta)\int_\Gamma \mathbf{H}_2 \times n_2 \cdot \mathbf{E}_2 ds + \gamma \int_\Gamma \mathbf{H}_1 \times n_2 \cdot \mathbf{E}_2 ds + \delta \int_\Gamma \mathbf{E}_1 \times n_2 \cdot \mathbf{H}_2 ds
\end{aligned}$$

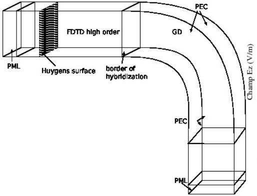
Then, by using $n_1 = -n_2$, to preserve the energy we must have the conditions :

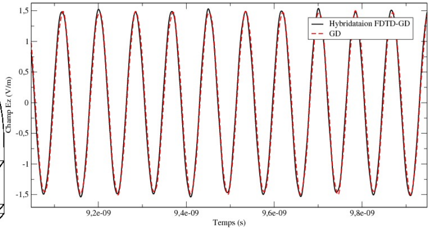
$$\begin{aligned}
1-\alpha+\beta =& 0 \\
1-\gamma+\delta =& 0 \\
\alpha+\delta =& 0 \\
\beta+\gamma =& 0
\end{aligned}$$

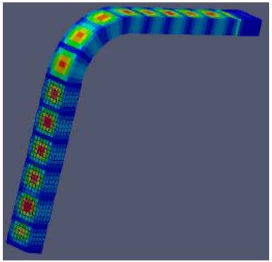
# Numerical example : validation of the hybridization principle

## Propagative mode inside a curved guide for a long duration of observation

- the guide is divided into two parts on which a FEM and a DG scheme is applied respectively
- a comparison between hybrid FEM/DG and DG methods is done



# Hybrid TLM-3D approach

## Numerical study : Mathematical problem

$$
\left\{
\begin{array}{l}
\varepsilon \dfrac{\partial E}{\partial t} + \sigma E + J = \nabla \times H \\
\mu_0 \frac{\partial H}{\partial t} = -\nabla \times E \\
\\
E(t=0,.) = H(t=0,.) = 0 \quad \text{on } \Omega \\
\forall t \in ]0,T[, n \times E(t,x) = 0 \quad \forall x \in \partial\Omega \\
\\
\text{On } \Omega_k, \quad \forall k \in \{1,...,H\} : \\
\quad L_k \dfrac{\partial I_k}{\partial t} + R_k I_k = -\dfrac{\partial V_k}{\partial \ell} - E \cdot u_k \\
\\
\text{On } \Omega_k, \quad \forall k \in \{1,...,N\} : \\
\quad C_k \dfrac{\partial V_k}{\partial t} + G_k V_k = -\dfrac{\partial I_k}{\partial \ell} \\
\\
I_k(t=0,.) = V_k(t=0,.) = 0 \;\; \text{on } \Omega_k, \forall k \in \{1,...,H\} \\
+ \text{ boundaries conditions on } JU_m, \forall m \in \{1,...,M\}
\end{array}
\right.
$$

# Hybrid TLM-3D approach

## Numerical study : Constraints to have a well-posed hybrid problem

The problem can be rewritten

$$\frac{U^{n+1} - U^n}{\Delta t} = AU^n + F^n,$$

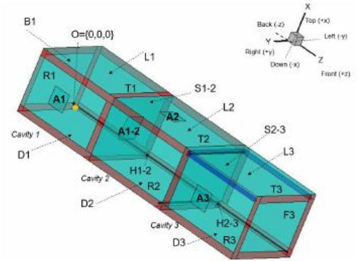
with $U^n = (E^n, H^{n-1/2}, I^{n-1/2}, V^n)$ and $A = \begin{pmatrix} A_E & A_{EI} \\ A_{IE} & A_I \end{pmatrix}$ where $A_E$ and $A_I$ are the discrete operators that approximate the differential operators $\begin{pmatrix} 0 & -\nu\nabla\times \\ \nu\nabla\times & 0 \end{pmatrix}$ and $\begin{pmatrix} L^{-1}R & \nu\frac{\partial}{\partial\ell} \\ \nu\frac{\partial}{\partial\ell} & C^{-1}G \end{pmatrix}$, respectively. The discrete operators $A_{EI}$ and $A_{IE}$ approximate the coupling operators $\begin{pmatrix} \frac{1}{S\sqrt{\varepsilon_0 L}}u_\ell\mathcal{L} & 0 \\ 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} -\frac{1}{\sqrt{\varepsilon_0 L}}\gamma_{u_\ell} & 0 \\ 0 & 0 \end{pmatrix}$, respectively. If we assume that these operators satisfy

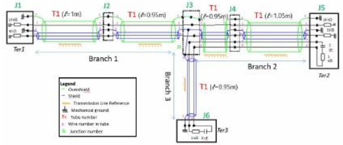
$$\langle A_{IE}E, I\rangle = -\langle A_{EI}I, E\rangle.$$

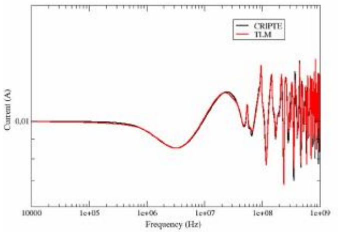
The hybrid TLM-3D problem is a well-posed problem

# Hybrid TLM-3D approach

## Validation

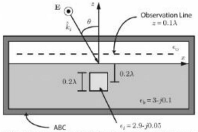


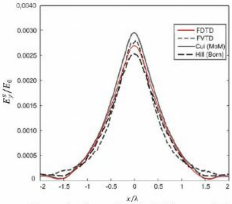



# New work in Maxwell-TD : Hybrid FVTD ST/NST method

## SER of buried objects located inside a soil (L-M. Mazzolo thesis)




Dustin Isleifson, Ian Jeffrey, Lotfollah Shafai, Joe Lovetri and David G. Barber, "An Efficient Scattered-Field Formulation for Objects in Layered Media Using the FVTD Method", in *IEEE Transactions on Antennas and Propagation*, vol. 59, no. 11, pp. 4162-4170, Nov. 2011, doi: 10.1109/TAP.2011.2164198




*Champs électriques rétrodiffusés (champ proche)*

*Définition de :* $H'_y = \frac{[illegible]}{\varepsilon'_r}$

$$\left.\begin{aligned} \frac{\partial E_y}{\partial z} &= \mu_0 \frac{\partial H_x}{\partial t} \\ \frac{\partial H_x}{\partial z} &= \sigma E_y + \varepsilon_0(\varepsilon'_r - \varepsilon_{r0} sin^2\theta)\frac{\partial E_y}{\partial t} \\ H_z &= Y_0\sqrt{\varepsilon_{r0}}\sin\theta E_y \end{aligned}\right\} \text{(TE)} \qquad \left.\begin{aligned} \frac{\partial E_x}{\partial z} &= -\sigma\frac{\mu_0}{\varepsilon_0}H'_y - \mu_0(\varepsilon'_r - \varepsilon_{r0} sin^2\theta)\frac{\partial H'_y}{\partial t} \\ \frac{\partial H_y}{\partial z} &= \sigma E_x - \varepsilon'_r\varepsilon_0\frac{\partial E_x}{\partial t} \\ E_z &= -Z_0\sqrt{\varepsilon_{r0}}\sin\theta H'_y \end{aligned}\right\} \text{(TM)}$$

$$\frac{\partial H_y}{\partial t} = \frac{\sigma}{\varepsilon_0}H'_y + \varepsilon'_r\frac{\partial H'_y}{\partial t}$$

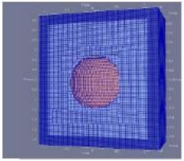
Avec : $\left| \begin{aligned} Y_0 &= \sqrt{\varepsilon_0/\mu_0} \\ Z_0 &= 1/Y_0 \end{aligned}\right.$

# New work in Maxwell-TD

## Hybrid ST/NST finite volume approach : idea

Evaluation of RCS of PEC sphere (radius: $0.5m$) using :

**1.** Purely Cartesian grid

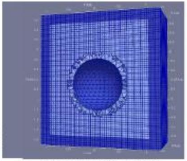


$59 \times 59 \times 64$ hexahedral cells

Equivalent mesh size:
$dx = dy = dz = 0.04\,m$

Tetrahedral cell edges in the surface object mesh $e = 0.08m$

**2.** Hybrid grid



$202\,717$ hexahedral cells
$21\,983$ tetrahedral cells

### Numerical modeling strategy

**Evaluation of RCS of PEC sphere (radius: $0.5m$)**

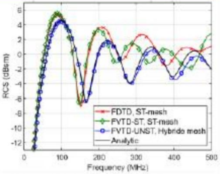
| | ST-mesh | | Hybrid mesh |
|---|---|---|---|
| **Scheme** | **FDTD** | **FVTD-ST** | **FVTD-UNST** |
| CPU time | 0m13.2s | 0m31.8s | **34m19.5s** |

Scheme providing solution closest to analytical:

- **FVTD-UNST** scheme but with **higher computational time**

**Hybridization of ST meshes with UNST meshes** associated with hybridization methods (**FDTD/FVTD-UNST** or **FVTD-ST/UNST**)

1 Core used

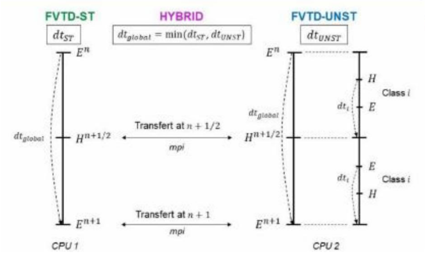



Processor Number: i7-1265U RAM: 16 Go
Performance Cores: 2 Cores, 4 Threads, 1.8 GHz Base
Efficient Cores: 8 Cores, 8 Threads, 1.3 GHz Base

# New work in Maxwell-TD

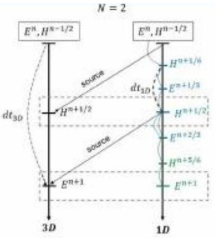
## Hybrid ST/NST finite volume approach : Exchange principle

### Hybrid method and Plane wave introduction



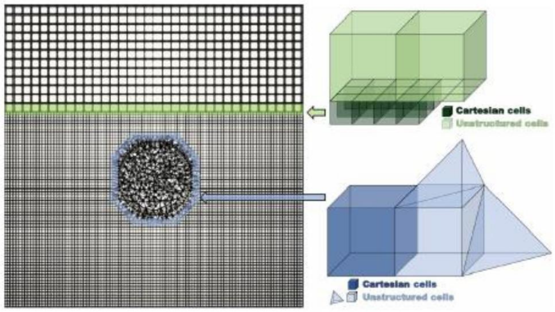

- Solving 3D Maxwell equations
- Solving 1D Maxwell's equations to evaluate source-related fields

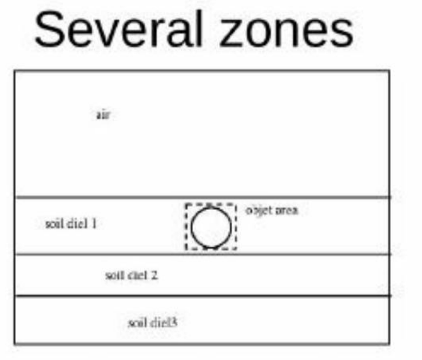
$dt_{1D} \leq dt_{3D}$ and $dt_{3D} = (2N-1) * dt_{1D}$
with $N$ an integer



# New work in Maxwell-TD

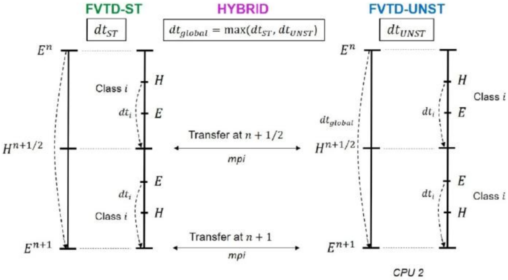
## Hybrid ST/NST finite volume approach : defintion of sub-domains



# New work in Maxwell-TD

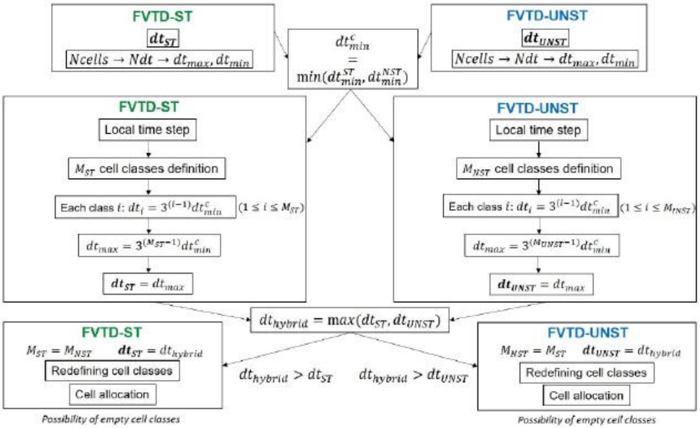
## Hybrid ST/NST finite volume approach : time exchanges




Each scheme use the same number of classes with the same timestep

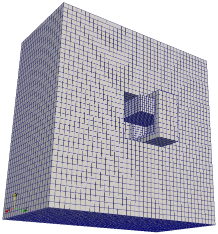
# New work in Maxwell-TD

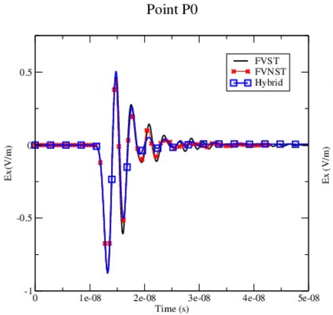
## Hybrid ST/NST finite volume approach : Evaluating time classes

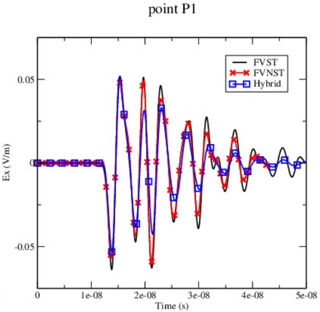


# New work in Maxwell-TD

## Hybrid ST/NST finite volume approach : first application






| FVTD NST | FVTD ST | Hybride |
|---|---|---|
| 2304s | 724s | 71s |

# New Numerical scheme in Maxwell-TD

## Compatible Operator Discrete method (V. Ritzenthaler thesis)

- Context : FDTD generalization. Possibility to introduce curved geometry and local refinements in FDTD.
- Mathematical model :

$$\partial_t \underline{B} - \mathsf{CURL}\ \underline{E} = 0, \qquad \underline{D} = \mathsf{H}_\varepsilon \underline{E}, \qquad (\mathsf{H}_\varepsilon)_{i,j} := \int_\Omega \varepsilon \ell_i(\mathbf{x})\ \ell_i(\mathbf{x})\ d\mathbf{x},$$

$$\partial_t \underline{D} + \mathsf{CURL}^T \underline{H} = -\underline{J}, \qquad \underline{H} = \mathsf{H}_{\mu^{-1}} \underline{B}. \qquad (\mathsf{H}_{\mu^{-1}})_{i,j} := \int_\Omega \frac{1}{\mu} h_i(\mathbf{x})\ h_j(\mathbf{x})\ d\mathbf{x}.$$

- Physical models : perfectly metallic, dielectric
- Meshes : unstructured meshes with local refinements
- Advantage : good approach to take into account complex structure with a low computational cost
- weak point of the method : local matrix to inverse (MUMPS solver)

# New Numerical scheme in Maxwell-TD

## Compatible Operator Discrete method : Topological Law

$$\begin{cases} \varepsilon \partial_t \mathbf{E} - \nabla \times \mathbf{H} = -\mathbf{J} \\ \mu \partial_t \mathbf{H} + \nabla \times \mathbf{E} = 0 \end{cases}$$

Relations de conservation

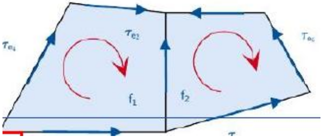
$$\begin{cases} \partial_t \mathbf{D} - \nabla \times \mathbf{H} = -\mathbf{J} \\ \partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0 \end{cases}$$

Relations constitutives

$$\mathbf{D} = \varepsilon \mathbf{E},\ \mathbf{B} = \mu \mathbf{H}$$

Topological Laws

$$\begin{array}{l} \partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0 \\ \partial_t \mathbf{D} - \nabla \times \mathbf{H} = -\mathbf{J} \end{array} \longrightarrow \begin{array}{l} \partial_t \underline{\mathrm{B}} + \mathrm{ROT}\ \underline{\mathrm{E}} = 0 \\ \partial_t \underline{\tilde{\mathrm{D}}} - \widetilde{\mathrm{ROT}}\ \underline{\tilde{\mathrm{H}}} = -\underline{\tilde{\mathrm{J}}} \end{array}$$

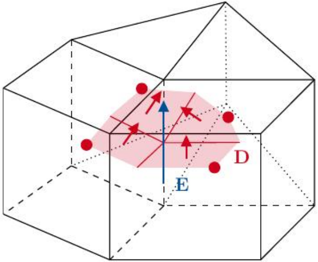



with ROT operator a matrix those termes are 0, 1 and -1

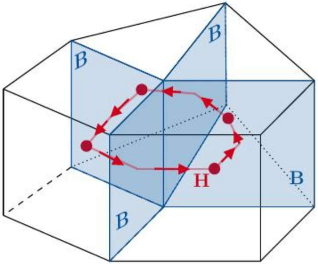
# CDO scheme : Topological equations

## location of unknowns

$$\mathbf{D} = \varepsilon \mathbf{E}$$




$$\mathbf{H} = \frac{1}{\mu} \mathbf{B}$$



# New Numerical scheme in Maxwell-TD

## Compatible Operator Discrete method : constitutive relations

$$\mathbf{D} = \varepsilon \mathbf{E} \qquad \qquad R_{\tilde{\mathcal{F}}}(\mathbf{D}) = H_\varepsilon R_{\mathcal{E}}(\mathbf{E})$$

$$\mathbf{H} = \frac{1}{\mu}\mathbf{B} \quad \cdots\cdots\blacktriangleright \quad R_{\tilde{\mathcal{E}}}(\mathbf{H}) = H_{\mu^{-1}} R_{\mathcal{F}}(\mathbf{B})$$

$$(H^c_\varepsilon)_{e_1,e_2} := \int_c \varepsilon\, \boldsymbol{\ell}_{e_1} \cdot \boldsymbol{\ell}_{e_2}, \quad e_1, e_2 \in E$$

$$(H^c_{\mu^{-1}})_{f_1,f_2} := \int_c \frac{1}{\mu}\, \boldsymbol{\ell}_{f_1} \cdot \boldsymbol{\ell}_{f_2}, \quad f_1, f_2 \in F$$

$$\frac{\underline{B}^{n+\frac{1}{2}} - \underline{B}^{n-\frac{1}{2}}}{\Delta t} + \mathrm{ROT}\ \underline{E}^n = 0$$

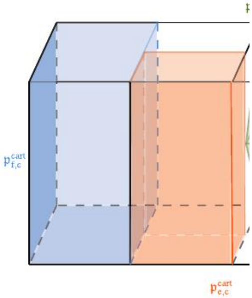
$$\underline{\tilde{H}}^{n+\frac{1}{2}} = H_{\mu^{-1}} \underline{B}^{n+\frac{1}{2}}$$

$$\frac{\underline{\tilde{D}}^{n+1} - \underline{\tilde{D}}^{n}}{\Delta t} - \widetilde{\mathrm{ROT}}\ \underline{\tilde{H}}^{n+\frac{1}{2}} = -\underline{\tilde{J}}^n$$

$$\underline{E}^{n+1} = H_\varepsilon^{-1} \underline{\tilde{D}}^{n+1}$$

# New Numerical scheme in Maxwell-TD

## Compatible Operator Discrete method : shape basis functions for cartesian cell

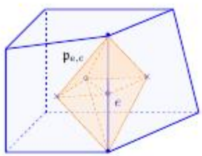


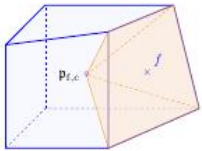

$$\boldsymbol{\ell}^{\text{cart}}_{\text{e,c}}(\boldsymbol{x}) := \begin{cases} |\text{e}|^{-1}\boldsymbol{\tau}_{\text{e}} & \text{si } \boldsymbol{x} \in \mathfrak{p}^{\text{cart}}_{\text{e,c}}, \\ 0 & \text{sinon}, \end{cases} \quad \text{and} \quad \boldsymbol{\ell}^{\text{cart}}_{\text{f,c}}(\boldsymbol{x}) := \begin{cases} |\text{f}|^{-1}\boldsymbol{n}_{\text{f}} & \text{si } \boldsymbol{x} \in \mathfrak{p}^{\text{cart}}_{\text{f,c}}, \\ 0 & \text{sinon}, \end{cases}$$

# New Numerical scheme in Maxwell-TD

## Compatible Operator Discrete method : shape function for polyhedric cell

$$\underline{\mathsf{L}}_{\mathcal{E}}(\mathbf{u})(\boldsymbol{x}) := \sum_{\mathsf{e}\in\mathsf{E}} \underline{\mathsf{u}}_{\mathsf{e}}\boldsymbol{\ell}_{\mathsf{e}}(\boldsymbol{x}) \qquad \underline{\mathsf{L}}_{\mathcal{F}}(\mathbf{v})(\boldsymbol{x}) := \sum_{\mathsf{f}\in\mathsf{F}} \underline{\mathsf{v}}_{\mathsf{f}}\boldsymbol{\ell}_{\mathsf{f}}(\boldsymbol{x})$$




$$\boldsymbol{\ell}_{\mathrm{f,c}}|_{\mathfrak{p}_{\mathrm{f',c}}} := \frac{\beta}{|\mathfrak{p}_{\mathrm{f,c}}|}\tilde{\mathbf{e}}_{\mathrm{c}}(\mathrm{f})\delta_{\mathrm{f,f'}} + \left(\underline{\underline{\mathrm{Id}}} - \beta\frac{\tilde{\mathbf{e}}_{\mathrm{c}}(\mathrm{f'})\otimes\mathbf{f'}}{|\mathfrak{p}_{\mathrm{f',c}}|}\right)\frac{\tilde{\mathbf{e}}_{\mathrm{c}}(\mathrm{f})}{|\mathrm{c}|} \quad \forall \mathrm{f},\mathrm{f'}\in\mathrm{F_c},$$

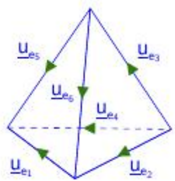
$$\boldsymbol{\ell}_{\mathrm{e,c}}|_{\mathfrak{p}_{\mathrm{e',c}}} := \frac{\beta}{|\mathfrak{p}_{\mathrm{e,c}}|}\tilde{\mathbf{f}}_{\mathrm{c}}(\mathrm{e})\delta_{\mathrm{e,e'}} + \left(\underline{\underline{\mathrm{Id}}} - \beta\frac{\tilde{\mathbf{f}}_{\mathrm{c}}(\mathrm{e'})\otimes\mathbf{e'}}{|\mathfrak{p}_{\mathrm{e',c}}|}\right)\frac{\tilde{\mathbf{f}}_{\mathrm{c}}(\mathrm{e})}{|\mathrm{c}|} \quad \forall \mathrm{e},\mathrm{e'}\in\mathrm{E_c},$$

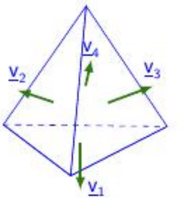
$$\tilde{\mathbf{e}}_{\mathbf{c}}(\mathrm{f}) = |\tilde{\mathrm{e}}_{\mathrm{c}}(\mathrm{f})|\boldsymbol{\tau}_{\tilde{\mathrm{e}}_{\mathrm{c}}(\mathrm{f})},\ \tilde{\mathbf{f}}_{\mathbf{c}}(\mathrm{e}) = |\tilde{\mathrm{f}}_{\mathrm{c}}(\mathrm{e})|\boldsymbol{n}_{\tilde{\mathrm{f}}_{\mathrm{c}}(\mathrm{e})},\ \mathbf{e} = |\mathrm{e}|\boldsymbol{\tau}_{\mathrm{e}} \quad \text{et} \quad \mathbf{f} = |\mathrm{f}|\boldsymbol{n}_{\mathrm{f}}.$$

# New Numerical scheme in Maxwell-TD

## Compatible Operator Discrete method : shape functions fo tetraedric cell

$$\underline{L}_{\mathcal{E}}^{W}(\underline{u})(x) := \sum_{e\in E} \underline{u}_e \underline{\ell}_e^{NED}(x) \qquad \underline{L}_{\mathcal{F}}^{W}(\underline{v})(x) := \sum_{f\in F} \underline{v}_f \underline{\ell}_f^{RTN}(x)$$



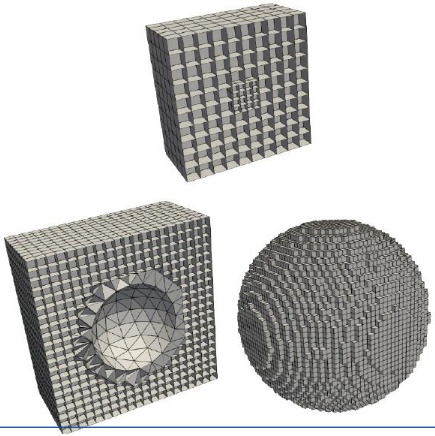



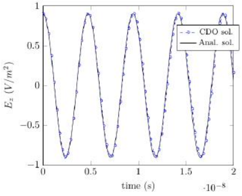

$$\boldsymbol{\ell}_{\mathrm{e,c}} = \frac{(\boldsymbol{x} - \boldsymbol{x}_{\mathrm{e(e)}}) \times \boldsymbol{\tau}_{\mathrm{e(e)}}}{\boldsymbol{e} \cdot \big((\boldsymbol{x}_{\mathrm{e}} - \boldsymbol{x}_{\mathrm{e(e)}}) \times \boldsymbol{\tau}_{\mathrm{e(e)}}\big)} \quad \text{et} \quad \boldsymbol{\ell}_{\mathrm{f,c}}(\boldsymbol{x}) = \iota_{\mathrm{f,c}} \frac{\boldsymbol{x} - \boldsymbol{x}_{\mathrm{v(f)}}}{d|\mathrm{c}|}.$$

# New Numerical scheme in Maxwell-TD

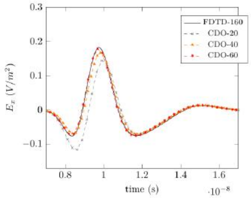
## Compatible Operator Discrete method : Example on local refinement

*5.1. Mesh with a local-refinement*





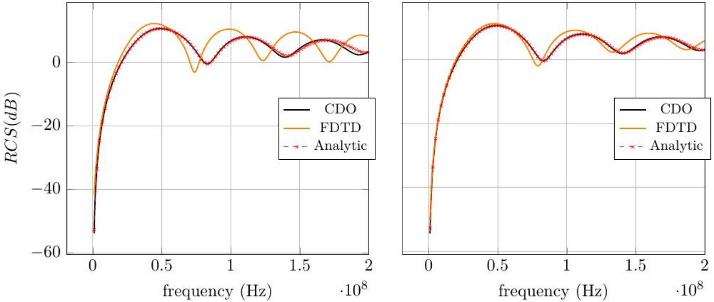

gure 7: Numerical solutions of the CDO method at point (0.4, 0.4, 0.4).



# New Numerical scheme in Maxwell-TD

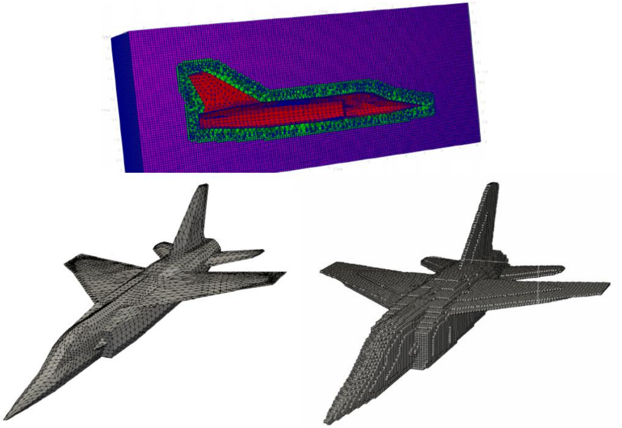
## Compatible Operator Discrete method : Application to SER of sphere

Results for $\lambda/10$ and $\lambda/20$ cell's size in the mesh



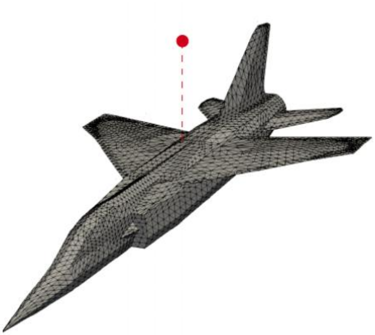
# New Numerical scheme in Maxwell-TD

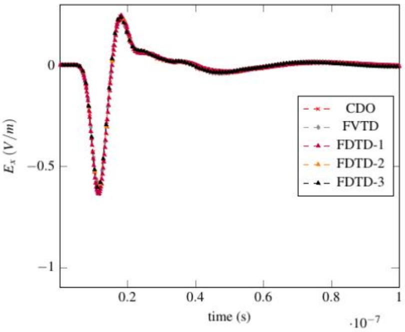
## Compatible Operator Discrete method : Hybrid mesh



# New Numerical scheme in Maxwell-TD

## Compatible Operator Discrete method : Hybrid mesh (point 1)

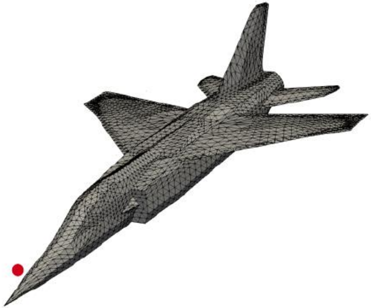


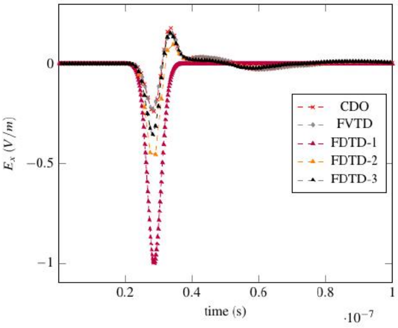


# New Numerical scheme in Maxwell-TD

## Compatible Operator Discrete method : Hybrid mesh (point 2)

**CPU time comparison** CDO : 30mn, FDTD-3 : more than 2h





# Conclusions and outlook

## Conclusions

**On numerical scheme : Maxwell-TD**

- allows us to introduce easily several scheme ;
- capitalize on the development ;
- possibility to share some physical models (wires, aperture, dispersive material, ...) with low development ;
- implementation for schema hybridization simplified

**On the solution mesh proposed**

- enables a mesh solution with a lot of cartesian cells ;
- allows to introduce locally unstructured cells in the mesh to take into account curved geometries ;
- authorizes to use and to couple several mesh zones in a global computational domain.

# Conclusions and outlook
## outlook

Several possibilities
Priority works for me

- In progress TAMI and DIAANE research projets ;
- In progress (V. Mouysset) study of a block structured mesher ;
- Define an efficient multi-mesh strategy with a maximum of cartesian zones ;
- Development for each existing method in Maxwell-TD of a version for cartesian mesh ;
- Make hybridation studies for FVTD-FDTD, CDO-FDTD, CDO-FVTD, CDO-TLM ;
- Make hybridations between scheme with low order and scheme with high order like FVTD-GD, FEM-FDTD, SD-FVTD (idea : reduce dispersive and dissipative error without increasing the CPU time)

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**Thank you for your Attention ! ! !**

**some questions ?**

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