Optimization of equivalent circuit models for metal-patterned metasurfaces.

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Contents

Introduction

Stealth objective.

- Approaches used in the literature for the attenuation of specular reflection:
  - *Diffraction* in a chosen direction,
  - *Deviation* of the incident waves,
  - *Attenuation* of the reflected wave amplitude.

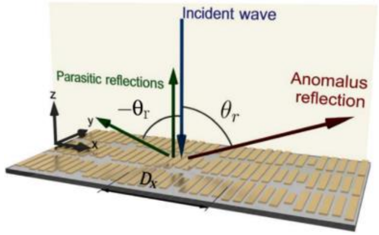


Fig. Diaz (2017)

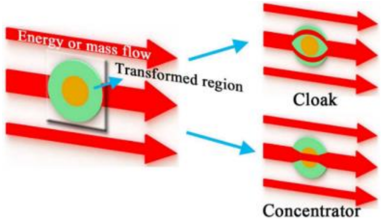


Fig. Yang (2024)

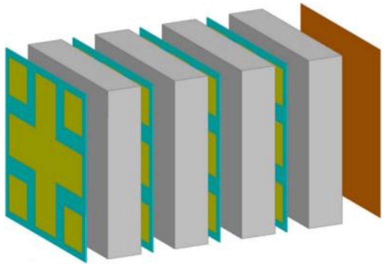


Fig. Li (2022)

- Absorbing coating (*lossy* materials):
  - Dielectric and ferromagnetic materials.

- Properties limitation of classical material:
  - Frequency range of absorption and material thickness.

➢ Use of metamaterials (**metasurfaces**): material designed to exhibit specific electromagnetic properties.

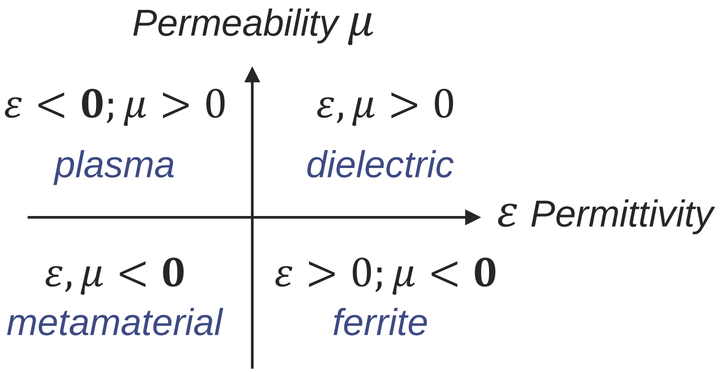
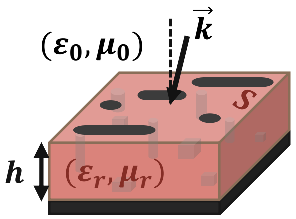


Fig. Electromagnetic media classification.

- Baseline: absorbing materials.
  - Homogeneous and isotropic.
  - *Intrinsic* properties: permittivity $\varepsilon_r \in \mathbb{C}$,
    permeability $\mu_r \in \mathbb{C}$.
  - *Electromagnetic* properties: Surface impedance $Z = j\sqrt{\mu/\varepsilon}\tan(\omega\sqrt{\mu\varepsilon}\,h/c_0)$,
    Reflection coefficient $R = \frac{Z-1}{Z+1}$,
    Radar cross-section $\sigma_{\mathrm{RCS}} = f(R, S)$.



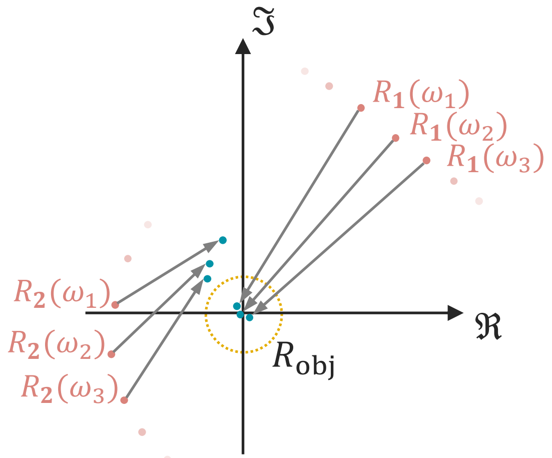
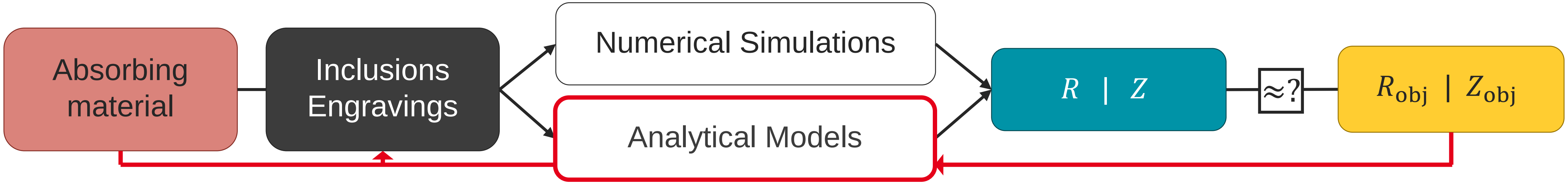


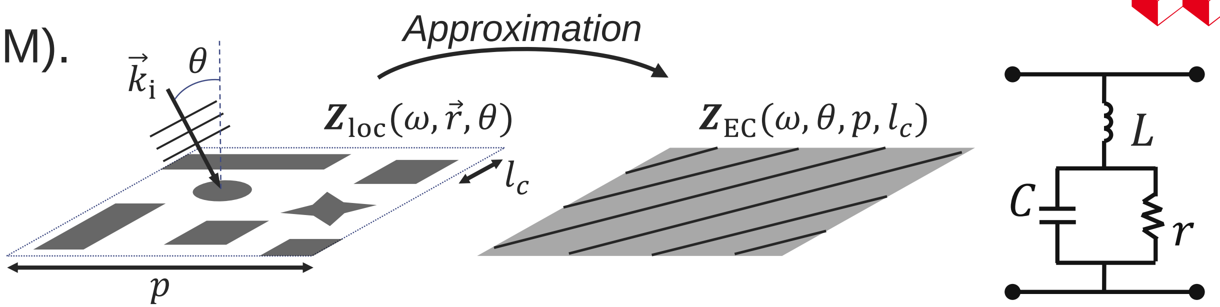
Fig. Complex plane.

- Modification of substrate properties by **inclusions**.
  - Metallic pattern with no thickness,
  - Periodic and dense distribution.

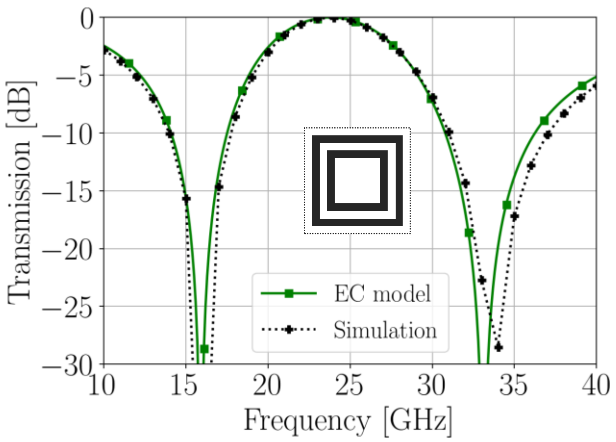
➢ Optimization of inclusion geometry: numerical or analytical approaches.



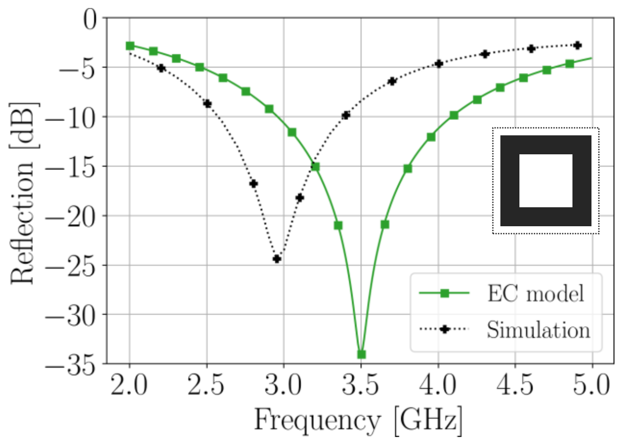
- Analytical approach: **Equivalent Circuit Model** (ECM).
  - Initial conditions:
    - *Infinite* plate length with *periodic cells* of size p,
    - Wavelength $\lambda \gg p \gg l_c$ characteristic length.



- Examples:
  - Double square loops without substrate.
    - $\lambda/p \in [1.5, 6]$
    - $p = 5$ mm,
    - $g = 0.5$ mm,
    - $w = 0.15$ mm.



  - Square loops with substrate.
    - $\lambda/p \in [6, 15]$
    - $p = 10$ mm,
    - $g = 3.2$ mm,
    - $w = 2.1$ mm.



- Sources of discrepancies:
  - Use of equivalent circuit models outside their **validity domain**?
  - Errors due to **external factors** affecting the modeling of metallic patterns?

➢ <u>Prior to optimization:</u> identification of potential sources of discrepancies and errors.

Contents

Equivalent circuit modeling of metasurfaces

- *Theoretical* study of the electromagnetic response of an **infinite array of parallel metallic strips** under an incident electromagnetic wave.
  References: Lamb (1898), Macfarlane (1947), Wait (1951), Marcuvitz (1951), …
  - Hypothesis: $\lambda \gg p \gg l_c$,
  - Impedance derivation in *quasi-static* regime: $\vec{E}_t = Z_s(\vec{n} \times \vec{H}_t)$.

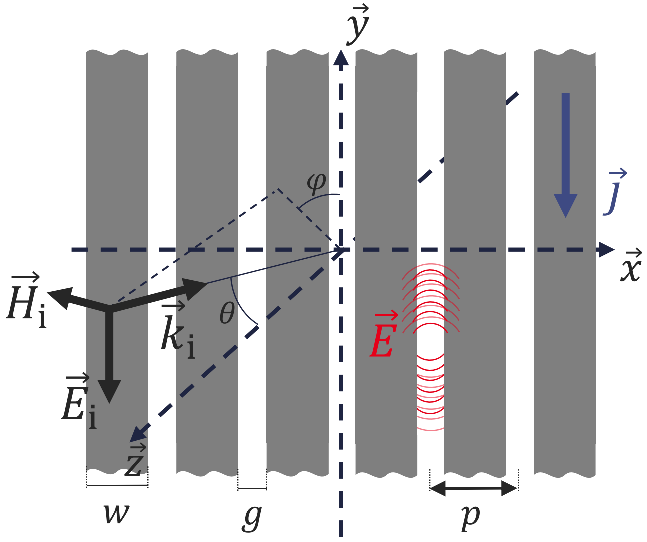
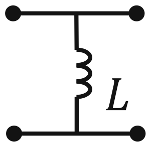
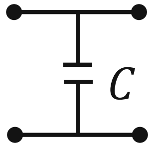


Fig. Array of parallel metallic strips.



1) Inductive case $[\vec{E}_\mathrm{i} \,/\!/\, \vec{y}]$

$$Z_{\mathrm{TE}} = jX(p, w, \lambda, \theta)\cos(\theta), \\ = j\omega L = jF(p, w, \lambda, \theta).$$

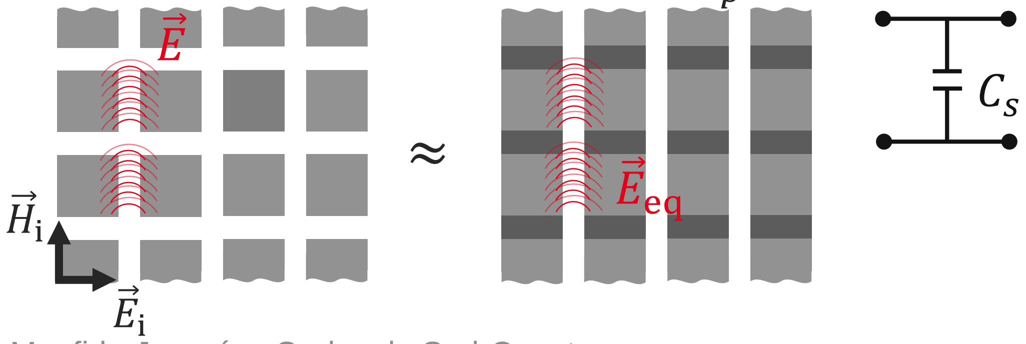


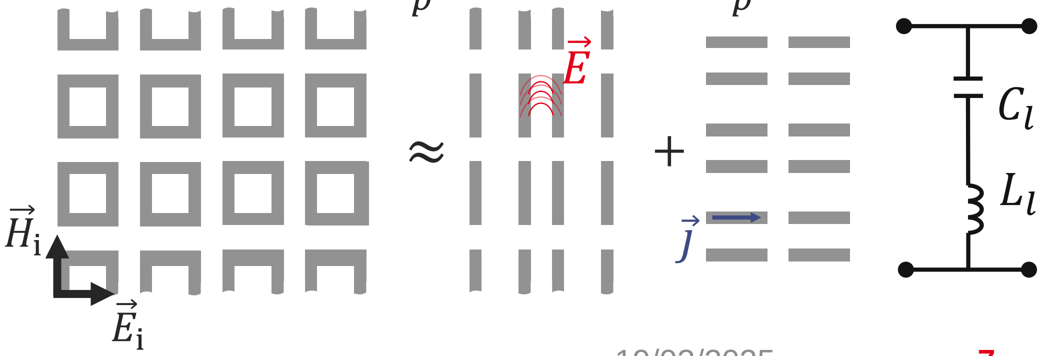
2) Capacitive case $[\vec{E}_\mathrm{i} \,/\!/\, \vec{x}]$

$$Z_{\mathrm{TE}} = \cos(\theta)\,/jB(p, g, \lambda, \theta), \\ = 1/j\omega C = 1/4jF(p, g, \lambda, \theta).$$

- where $F(p, l_c, \lambda, \theta) = p/\lambda \left[\log\left(1/\sin\left(\frac{\pi l_c}{2p}\right)\right) + G(p, l_c, \lambda, \theta)\right]$, with G a corrective term.

- Examples of patterns derived from the initial configuration:
  - Square metallic patches: $C_s = \frac{p-g}{p} C$.
  - Square loops: $C_l = \frac{p-g}{p} C$ and $L_l = \frac{p-g}{p} L$.





Equivalent circuit modeling of metasurfaces

- Validity domain of the ECM
  - Initial configuration: infinite parallel strips.
  - Lengths and frequency range of the study:
    $$[1\text{GHz}, 10\text{GHz}] \rightarrow \left\{\frac{\lambda}{w}, \frac{\lambda}{g}\right\} \in [10{,}100]$$
  - Reminder: $\lambda \gg p \gg l_c \in \{g, w\}$.
    - **Inductive effects** correctly captured with the equivalent circuit model,
    - **Capacitive effects** well approximated for sufficiently large w.

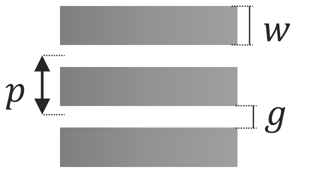


Fig. **Inductive** configuration

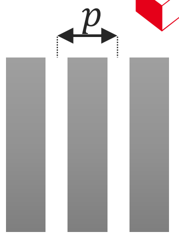


Fig. **Capacitive** configuration

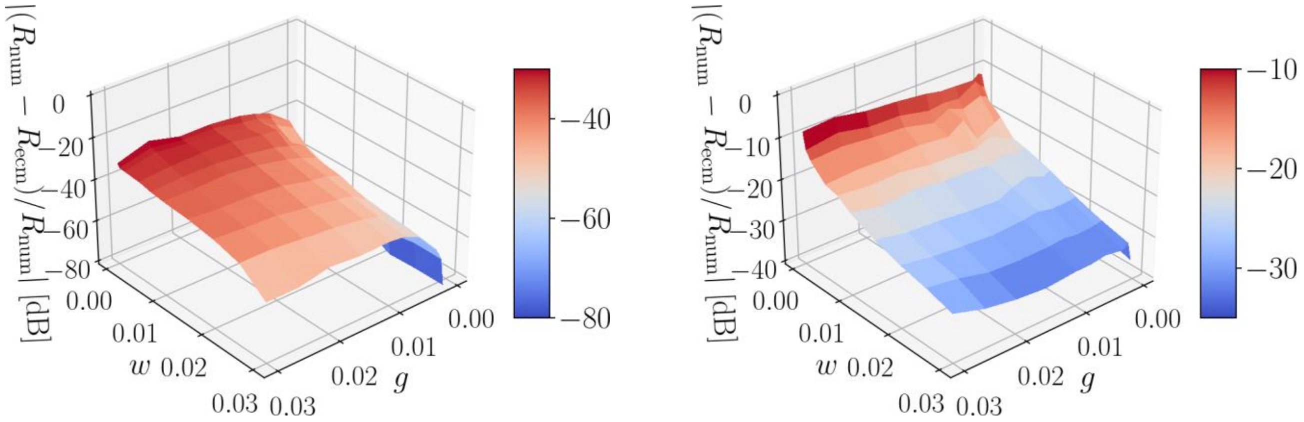
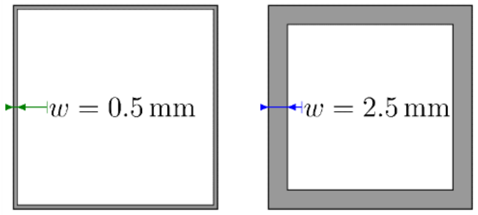
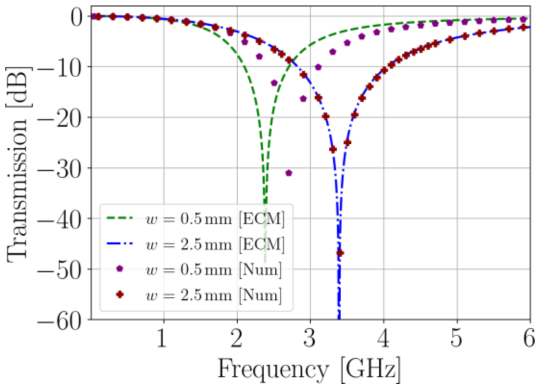


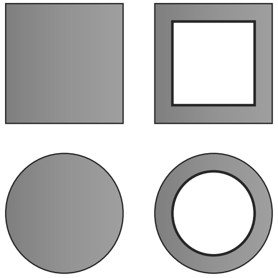
Fig. Relative error between simulations and EC modeling for the *inductive* (left) and *capacitive* (right) configuration.

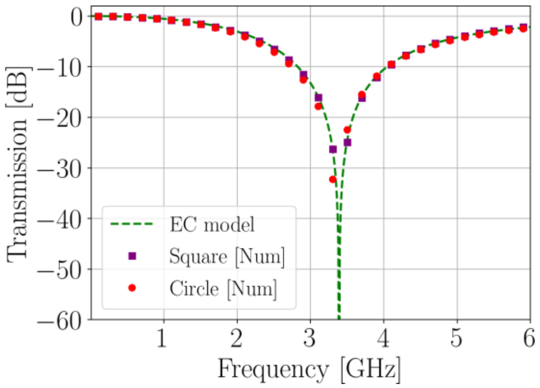
- Straightforward application on a square-loop surface:





- Remark on shape: square versus circle [Deng, 2024].





Equivalent circuit modeling of metasurfaces

Metasurface modeling: material (substrate) with surface inclusions.

- Substrate **permittivity**: capacity modification $\mathbf{1} \times B(\dots) \rightarrow \boldsymbol{\varepsilon}_{\mathbf{eff}} \times B(\dots)$.
  - General correction [Munk, 2000] : $\varepsilon_{\text{eff}} = \frac{(\varepsilon_0+\varepsilon_r)}{2}$
  - Specific correction [Costa, 2014; Ferreira, 2015] : $\varepsilon_{\text{eff}} = \frac{(\varepsilon_0+\varepsilon_r)}{2} + f(\boldsymbol{p}, \boldsymbol{g}, \boldsymbol{w}, \dots)$.
- Substrate **height**: effect of a metal ground.
  - Specific correction [Ferreira, 2015] : $\varepsilon_{\text{eff}} = \frac{(\varepsilon_0+\varepsilon_r)}{2} + f(p, g, w, \boldsymbol{h})$.
- *A posteriori* **correction** of the equivalent circuit:
  - Square slot FSS [Ferreira 2015] : $B_C = \varepsilon_{\text{eff}} \left(\mathbf{1.75}\, B_{c_1} + \mathbf{0.6}\, B_{c_2}\right)$
    with $B_{c_1} = 4\, F(p, d, \lambda)$, and $B_{c_2} = 4\, F(d - s, s, \lambda)$.
  - Cross-square resistive patterns [Li, 2022] :
    Resistivity : $r = \boldsymbol{\zeta}\, R_1 \frac{\sum S_{\text{tot}}}{\sum S_{\text{res}}}$, with $\boldsymbol{\zeta} \in [1.05, 1.11]$

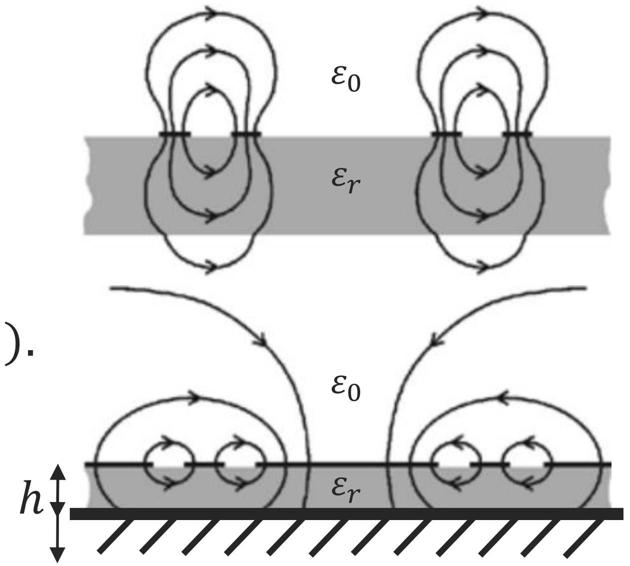


Fig. Sketch of electric-field lines [Baena *et al.* 2005]

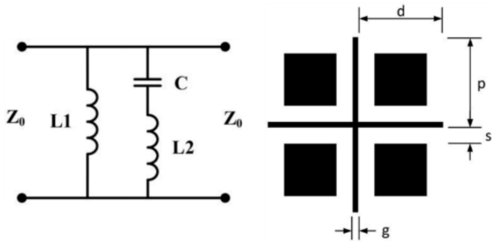


Fig. Square slot [Ferreira *et al.*, 2015]

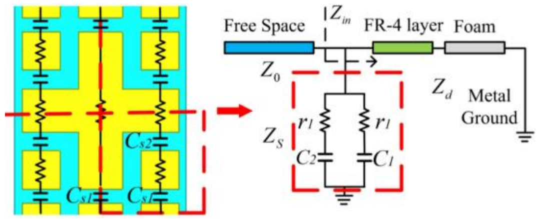
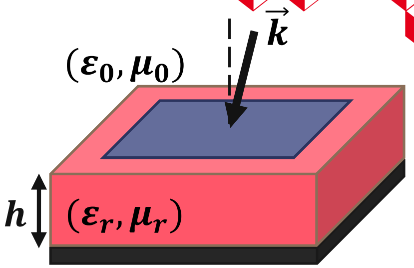


Fig. Cross-square resistive pattern [Li *et al.*, 2022]

Contents

Metasurface absorption optimization



Optimization with **1 degree of freedom**.

- Configuration:
  - Ferromagnetic material: $h = 2.5$ mm with known properties (ε_r, μ_r).
  - Frequency selective surface (FSS): square patches $\left(Z_{\text{FSS}}(p, g)\right)$.
  - Total surface impedance: $Z_{\text{tot}} = Z_{\text{mat}} \mathbin{/\!/} Z_{\text{FSS}}$.

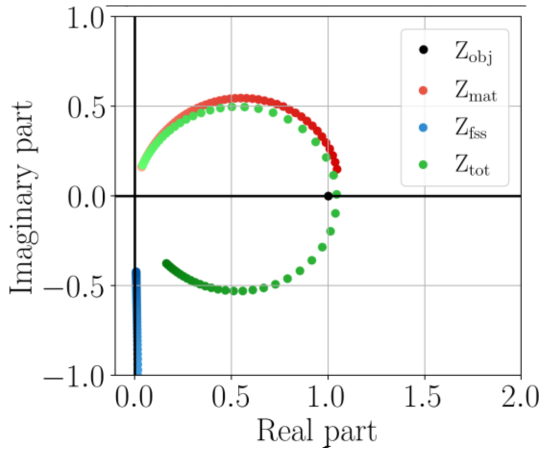


Fig. Impedance in the complex plane between 1 GHz and 6 GHz.

- Optimization at $f_{\text{tar}} = 3.5$ GHz:
  - Periodic-cell size: $p \in [3, 20]$ mm,
  - Objective function:

$$F_{\text{obj}}(g) = \left\| \frac{Z_{\text{tot}}(f_{\text{obj}}) - 1}{Z_{\text{tot}}(f_{\text{obj}}) + 1} \right\|_2$$

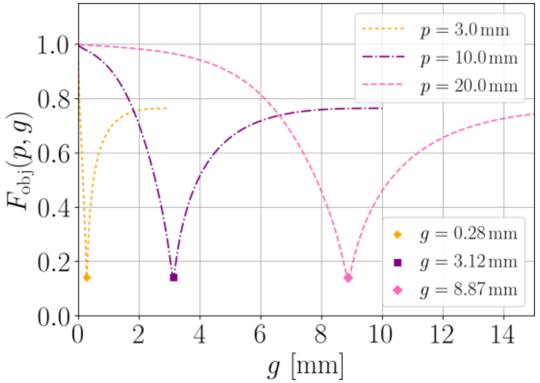


Fig. Minimization of the objective function for different values of p.

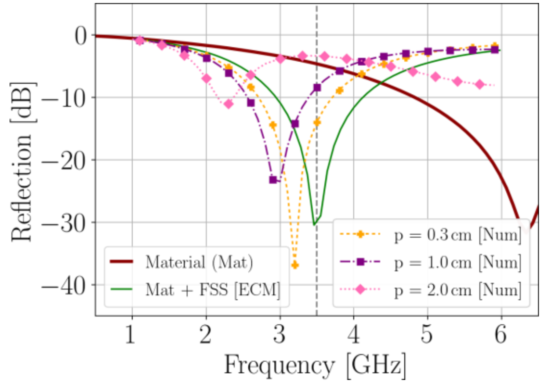
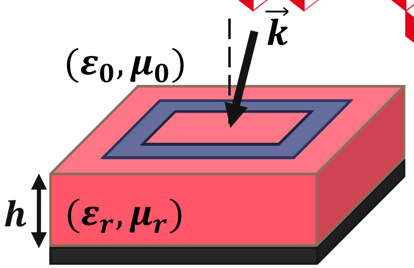


Fig. Reflection coefficient.

- Discrepancies:
  - Thickness of the substrate.
  - Inductive effects overlooked?

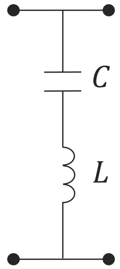
Metasurface absorption optimization

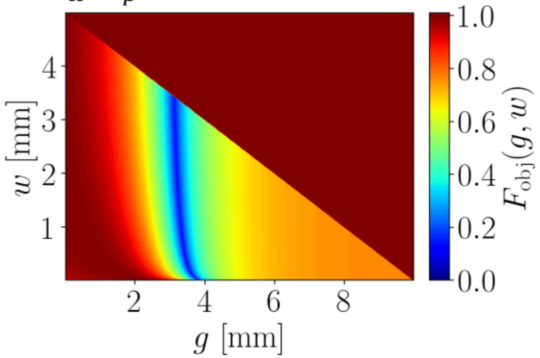


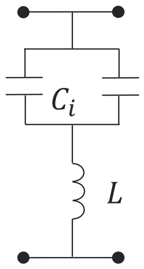
Optimization with **2 degrees of freedom**.

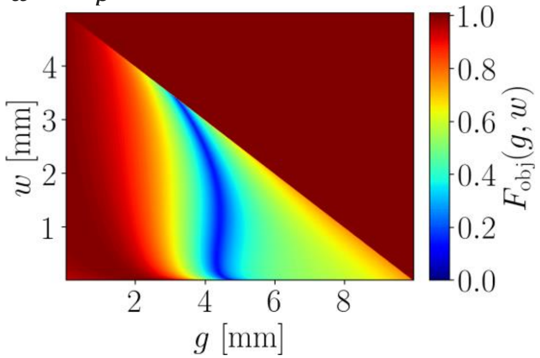
- Similar configuration:
  - Ferromagnetic material: $h = 2.5$ mm with known properties (ε_r, μ_r).
  - Frequency selective surface (FSS): square **loops** $\left(Z_{\text{FSS}}(p, g, w)\right)$.
  - Total surface impedance: $Z_{\text{tot}} = Z_{\text{mat}} /\!/ Z_{\text{FSS}}$.

- Optimization at $f_{\text{tar}} = 3.5$ GHz and $p = 10$ mm:
  - Objective function: $F_{\text{obj}}(g, w) = \left\| \frac{Z_{\text{tot}}(f_{\text{obj}}) - 1}{Z_{\text{tot}}(f_{\text{obj}}) + 1} \right\|_2$.
  - LC equivalent circuit:
    - $g_{\text{opt}} = 3.2$ mm, $w_{\text{opt}} = 2.1$ mm;
    - $C = \frac{1}{\omega} \frac{p-g}{p} B(p, g, \lambda, \theta)$.
  - LCC equivalent circuit:
    - $g_{\text{opt}} = 4.25$ mm; $w_{\text{opt}} = 2$ mm;
    - $C_i = \frac{1}{\omega} \frac{p-g-2w}{p} B_i(p, p-g-2w, \lambda, \theta)$.









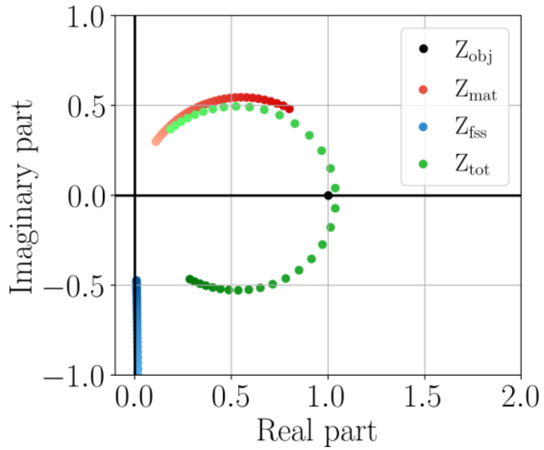


Fig. Impedance in the complex plane between 2 GHz and 5 GHz.

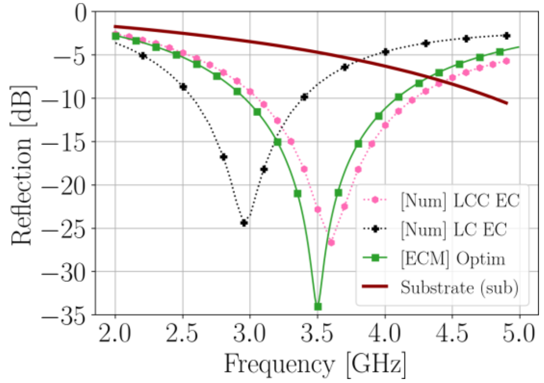
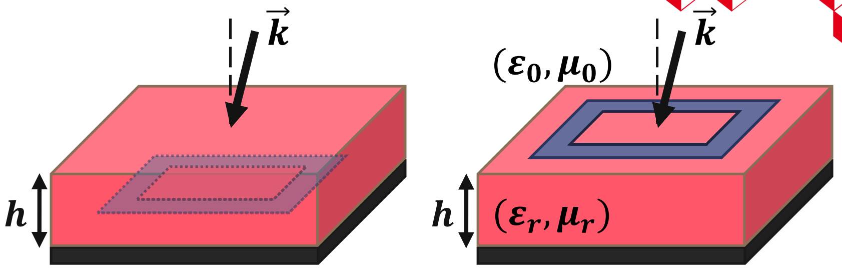


Fig. Reflection coefficient.

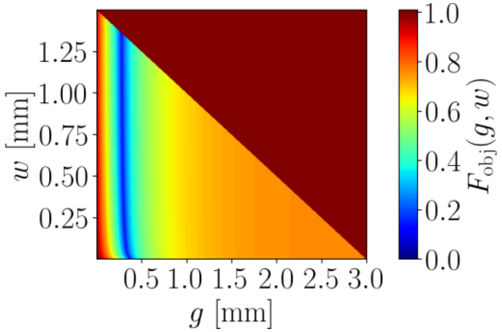
➢ Better modeling with the **LCC equivalent circuit**.



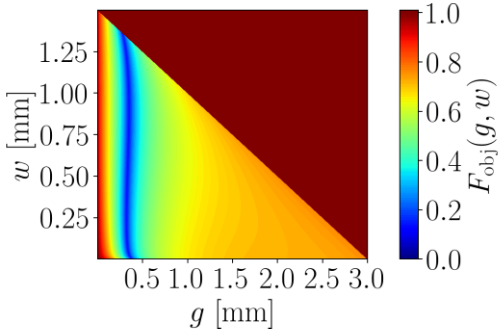
Optimization with **2.5 degrees of freedom**.

- Previous configuration with adjustments:
  - Square-loop FSS: $p = 3$ mm.
  - Configurations: [on] $h_{mot} = 2.5$ mm, [in] $h_{mot} = 1.25$ mm.

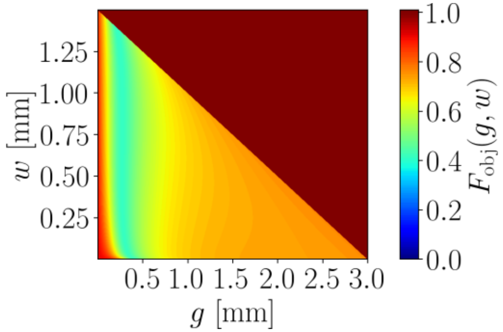
- Optimization at $f_{tar} = 3.5$ GHz:
  - LC [on]:



  - LCC [on]:



  - LCC [in]:



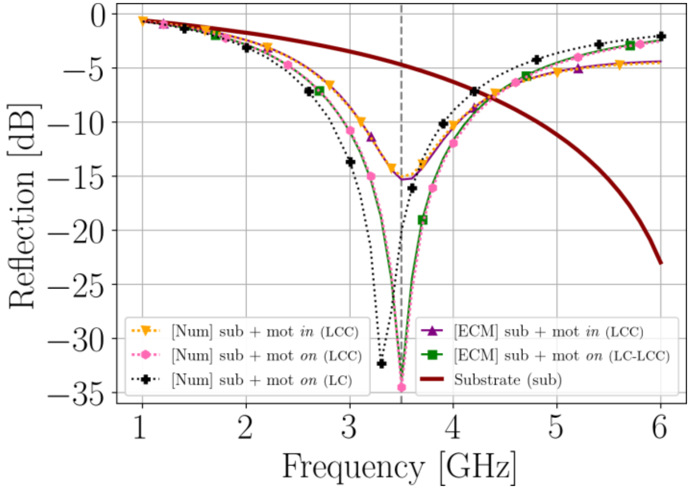


Fig. Metasurface reflection.

➢ Improved modeling with: $\boldsymbol{p} = \mathbf{3mm}$ and the **LCC EC**.

➢ Better absorption with the **[on] case** for the studied metasurface .

Remark: Infinite to finite problems.

- Finite square-loop metasurface [$p = 3$mm, $f = 3.5$ GHz].
- Study of the plate *boundaries* effects and the impedance *homogeneity*.

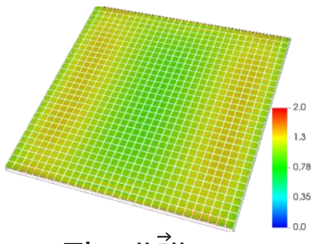


Fig. $||\vec{J}||$on a 9×9 cm plate.

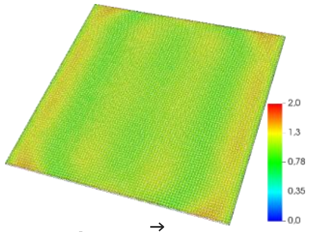


Fig. $||\vec{J}||$ on a 26×26 cm plate.

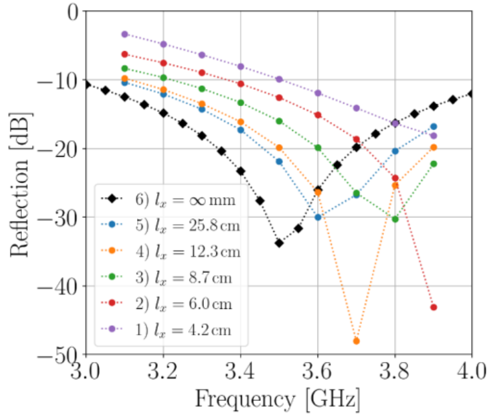


Fig. Infinite and finite metasurfaces.

Contents

Conclusion

- Main items covered:
  - Understanding of the **equivalent-circuit** analytical models.
  - Study of the **validity domain** of equivalent circuit models.
  - Successful **shape optimization** based on circuit models.

- Outlooks:
  - Extension of the absorption properties of metasurfaces:
    - Modeling of geometries with more **degrees of freedom**,
    - **Multilayer** metasurfaces with several FSS.
  - Consideration of **oblique-incidence** waves.
  - Application to **curved** metasurfaces [T. Bulteau, PhD].

Thank you

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